Institut de Mathématiques de Jussieu - Paris Rive Gauche
& Research Institute for Mathematical Sciences, Kyoto University

Paris Arithmetic Homotopic Galois Theory Days 2025

Program and schedule of the workshop

November 17-19, 2025 - Jussieu France

Org.: B. Collas, RIMS Kyoto University, Japan & A. Mézard, IMJ-PRG, FR & B. Enriquez, IRMA Strasbourg, France

Schedule

Talks are 60 minutes long. All talks take place at the Campus Jussieu ("Room AA-BB/FRR" is to be read "between tower AA and tower BB, at floor F, room RR"). Some ∼ 30 minutes long breaks between talks and extended lunch break allow informal exchanges between participants.

Monday - 17 Nov. ***Room 15-16/413***

9:15 .... *Welcome*

9:30 .... D. Shiraishi
On the ℓ-adic Galois analogue of multiple zeta values and polylogarithms

11:00 .... C. De Clercq
Artin upper motives

12:00 .... Lunch & Discussions

14:00 .... F. Orgogozo
Characteristic cycles, symmetric powers of curves, and determinant of étale cohomology

15:30 .... D. Corwin
Beyond Quadratic Chabauty: Tannakian Methods for CK Theory

Tuesday - Nov. 18 ***Room 15-16/413***

9:30 .... Y. Hoshi
The arithmetic fundamental groups of curves over local fields

11:00 .... J. Fresán
A construction of the polylogarithm motive

12:00 .... Lunch & Discussions

14:00 .... Y. Yatagawa*
Index formula of partially logarithmic characteristic cycles

15:00 .... Free Discussions

Wednesday - Nov. 19 ***Room 15-16/413***

9:30 .... S. Ishii
Galois actions on pro-p fundamental groups of once-punctured CM elliptic curves

11:00 .... H. Soré
On a Quillen adjunction between the categories of differential graded and simplicial coalgebras

12:00 .... Lunch & Discussions

14:00 .... H. Liu
Convolution groups of perverse sheaves on abelian varieties

15:30 .... D. Jarossay
$\mathcal{M}_{0,5}$: Towards the Chabauty-Kim method in higher dimensions

* *By Zoom, screening in the workshop room.*

"Restauration" - a selection

- Les petits pois, 3 Rue Linné, 75005 Paris - [Google Map]
- Brasserie Cosmo, 1 Rue des Écoles, 75005 Paris - [Google Map]
- Les Belles Plantes, 47 Rue Cuvier, 75005 Paris - [Google Map]
- Narro, 72 Rue du Cardinal Lemoine, 75005 Paris - [Google Map]
- Ippudo Saint-Germain, 714 Rue Grégoire de Tours, 75006 Paris - [Google Map]

A cafeteria at the corner of towers 16-26-25 on the "parvis" provides drinks for the session breaks.

Title & Abstracts

On the ℓ-adic Galois analogue of multiple zeta values and polylogarithms

Shiraishi Densuke (tokyo University of Science)

The ℓ-adic Galois analogues of multiple zeta values and polylogarithms are defined using noncommutative 1-cocycles arising from the Galois action on the pro-ℓ etale fundamental groupoid of $\mathbb{P}^1\backslash\{0,1,\infty\}$ with rational base points. These objects were first introduced by Wojtkowiak and are closely related to the Soule character, along with its polylogarithmic refinement—the generalized Soule character—formulated by Nakamura-Wojtkowiak.

In this talk, we will discuss the relations among ℓ-adic Galois multiple zeta values and functional equations of ℓ-adic Galois polylogarithms.

[NW12] H. Nakamura, Z. Wojtkowiak, Tensor and homotopy criteria for functional equations of ℓ-adic and classical iterated integrals. Non-abelian fundamental groups and Iwasawa theory, 258–310, London Math. Soc. Lecture Note Ser., **393**, Cambridge Univ. Press, Cambridge, 2012.

[NS25] H. Nakamura, D. Shiraishi, Landen's trilogarithm functional equation and ℓ-adic Galois multiple polylogarithms. Springer Proc. Math. Stat., **456**, Springer, Singapore, 2025, 237-262.

[S23] D. Shiraishi, Spence-Kummer's trilogarithm functional equation and the underlying geometry. preprint, https://arxiv.org/abs/2307.09414.

Charles De Clercq (Université Sorbone Paris Nord)

Studying geometry of varieties without rational points is a very challenging problem. In the last decades Chow motives have shown to be a powerful tool for this sake: motivic decompositions of projective quadrics and Severi-Brauer varieties provided numerous solutions to classical conjectures on quadratic forms and central simple algebras over fields.

In this talk, we work in the broader context of projective homogeneous varieties for reductive groups and start by reviewing the main properties of their integral motives, notably the Rost nilpotence principle. Following our recent works, we will introduce a new family of geometric motives built out of the Artin motives: the A-upper motives. Investigating the Tate subcategory generated by A-upper motives, we show how Artin-Tate motives can be used to describe motivic decompositions of projective homogeneous varieties, to obtain a classification of these motives and to solve classical geometric problems. We will then discuss how these techniques can be extended to other algebraic varieties over fields.

[KA11] N. Karpenko, Canonical dimension, *Proceedings of the international congress of mathematicians (ICM 2010)*, Hyderabad, India, August 19–27, 2010. Vol. II: Invited lectures (2011), 146–161.

[DKQ11] C. De Clercq, N. Karpenko, A. Quéguiner-Mathieu, A-upper motives of reductive groups (2024), https://arxiv.org/abs/2403.11030

Characteristic cycles, symmetric powers of curves, and determinant of étale cohomology

Fabrice Orgogozo (IMJ-PRG)

The geometry of the Abel-Jacobi morphism, from the symmetric product of a smooth connected projective curve X (over an algebraically closed field) to its Jacobian, is intimately connected with the Riemann–Roch theorem and plays a central role in geometric class field theory. For instance, the fact that, for n large enough, the morphism $Sym^n(X) \to Jac(X)$ is a locally trivial fibration in projective schemes (hence simply connected) implies that the fundamental group of $Jac(X)$ is isomorphic to that of $Sym^n(X)$, itself isomorphic to the abelianization of $\pi_1(X)$.

In a letter to Jean-Pierre Serre dated February 8, 1974, Pierre Deligne revisited this idea to study the determinant of étale cohomology — that is, the ε-factors — with coefficients in a possibly ramified rank-1 sheaf.

In this talk, we will present one of the ideas he also introduced in a 1980 seminar at the IHÉS, which makes it possible to treat the non-abelian case, namely that of a ramified sheaf of arbitrary rank, in the tame situation. Thanks to the theory of characteristic cycles developed by Alexander Beilinson and Takeshi Saito, we will interpret part of Deligne's calculations as a special case of a formula describing the characteristic cycle of a symmetric power of a curve – a formula whose analogue in characteristic 0 was previously stated by Gérard Laumon.

This is joint work with Joël Riou (Orsay).
(A similar result has been obtained independently by Will Sawin.)

[Bei17] Alexander Beilinson, « Constructible sheaves are holonomic », Selecta Math., 2017.

[Del01] Pierre Deligne, lettre à Jean-Pierre Serre (1974-2-8), dans « Gauss-Manin determinants for rank 1 irregular connections on curves » (2001), de Spencer Bloch et Hélène Esnault.

[Lau87] Gérard Laumon, « Correspondance de Langlands géométrique pour les corps de fonctions », Duke Mathematical Journal, 1987.

[OR25] Fabrice Orgogozo and Joël Riou, « Cycle caractéristique sur une puissance symétrique d'une courbe et déterminant de la cohomologie étale », Tunisian journal of mathematics (à paraître).

[Sai17] Takeshi Saitô, « The characteristic cycle and the singular support of a constructible sheaf », Inventiones mathematicae, 2017.

[Saw21] Will Sawin, « A geometric approach to the sup-norm problem for automorphic forms: the case of newforms on $GL_2(\mathbb{F}_q(T))$ with squarefree level », Proceedings of the London Mathematical Society, 2021.

Beyond Quadratic Chabauty: Tannakian Methods for CK Theory

David Corwin (Ben Gurion University)

Faltings' Theorem says that a hyperbolic curve has finitely many rational points; concretely, a nonsingular two-variable polynomial equation of degree at least 5 has finitely many rational solutions. The Quadratic Chabauty method has in recent years allowed us to provably find sets of rational points on previously inaccessible hyperbolic curves, but it still has limitations. Quadratic Chabauty is based on the more general non-abelian Chabauty method of Minhyong Kim (abbreviated as "Chabauty-Kim" or "CK"), but this method has rarely been applied outside the "quadratic" case.

We discuss a variety of work in progress, some joint with Ishai Dan-Cohen and/or Martin Lüdtke, which provide the tools for applying the method more generally. Our methods rely heavily on Tannakian categories of Galois representations or motives and their p-adic realizations and periods.

[Cor21] D. Corwin, Explicit Motivic Mixed Elliptic Chabauty-Kim (2021). https://arxiv.org/abs/2102.08371

[Cor25] D. Corwin, Tannakian Selemer vartieties (notes). https://math.bgu.ac.il/~corwind/files/research/TSV.pdf

The arithmetic fundamental groups of curves over local fields

Hoshi Yuichiro (RIMS Kyoto University)

In the paper published in 1999, Shinichi Mochizuki proved that every continuous isomorphism between the étale fundamental groups of hyperbolic curves over a sub-p-adic field arises from an isomorphism of hyperbolic curves. Recall that the main ingredients of his proof are various results in the study of p-adic Hodge theory.

The purpose of this talk is to explain an outline of a recent alternative proof of the above anabelian result in which we never apply such a result in p-adic Hodge theory. In the recent alternative proof, one avails oneself of a result on the infinitesimal Torelli problem for generalized Prym varieties, a classical extension result for homomorphisms between p-divisible groups, a classical result in the theory of deformation of ordinary semi-abelian varieties, and a classical result in the theory of degeneration of polarized abelian varieties. This is joint work with Yu Yang.

[Moc99] S. Mochizuki, The local pro-p anabelian geometry of curves, *Invent. Math.* **138**, no. 2 (1999), 319-423 DOI: 10.2996/kmj/1372337519

[HY25] Y. Hoshi and Y. Yang, The arithmetic fundamental groups of curves over local fields, *in preparation,* 2025

A construction of the polylogarithm motive

Javier Fresán (IMJ-PRG)

Classical polylogarithms give rise to a variation of mixed Hodge-Tate structures on the projective line minus three points, which is an extension of the symmetric power of the Kummer variation by a trivial variation. By results of Beilinson-Deligne, Huber-Wildeshaus and Ayoub, this polylogarithm variation has a lift to the category of mixed Tate motives, whose existence is proved by computing the corresponding spaces of extensions both in the Hodge and the motivic settings.

I will present a joint work with Clément Dupont, in which we construct the polylogarithm motive as an explicit relative cohomology motive.

[AY04] J. Ayoub, Motivic version of the classical polylogarithms, Polylogarithms, pp. 2563–2565, Oberwolfach Rep. 1 (2004), no. 4. https://user.math.uzh.ch/ayoub/Other-PDF/Polylogs.pdf

[DU24] C. Dupont, An introduction to mixed Tate motives, to appear in the proceedings of the IRMA Summer School on Motives and Arithmetic Groups (2024) https://arxiv.org/abs/2404.03770

[DF25] C. Dupont, J. Fresán, A construction of the polylogarithm motive, Épijournal de Géométrie Algébrique, volume 9 (2025), article no. 3. https://epiga.episciences.org/1527

Index formula of partially logarithmic characteristic cycles

Yatagawa Yuri (Institut of Science Tokyo)

In this talk, we consider the computation of the Euler characteristic of an ℓ-adic sheaf on a smooth variety in terms of ramification theory. Ramification theory in arithmetic geometry is a study of relations between cohomological invariants of a constructible sheaf and invariants measuring the ramification of the sheaf. Following Deligne's observation for an analogy between the wild ramification of ℓ-adicsheaves in positive characteristic and the irregular singularity of partial differential equations on a complex manifold, we consider a construction of an algebraic cycle for a constructible sheaf on the cotangent bundle which computes the Euler characteristic as the intersection number with the zero section by using ramification theory.

We construct an algebraic cycle on the logarithmic cotangent bundle with logarithmic poles along a subdivisor of the boundary for a smooth sheaf on a smooth variety following Kato's construction of the logarithmic characteristic cycle. After that, we give a formula for the Euler characteristic of the sheaf by using the algebraic cycle constructed with the invariants measuring the ramification of the sheaf.

*Talk by Zoom

[KA94] Kazuya KATO. Class field theory, $\mathcal{D}$-modules, and ramification on higher dimensional schemes, part I, Am. J. of Math. Vol. 116, No. 4 (1994), 757–784.

[SA09] Takeshi SAITO, Wild ramification and the characteristic cycle of an ℓ-adic sheaf, *J. Inst. Math. Jussieu.* **8** (2009), no. 4, 769–829.

[YA22] Yuri YATAGAWA, Singular support and characteristic cycle of a rank one sheaf in codimension 2 (2022), https://arxiv.org/abs/2206.02989

GALOIS ACTIONS ON PRO-p FUNDAMENTAL GROUPS OF ONCE-PUNCTURED CM ELLIPTIC CURVES

ISHII Shun (Keio University)

The Galois action on the pro-p fundamental group of $\mathbb{P}^1 - \{0, 1, \infty\}$ encodes rich information about pro-p extensions of the p-th cyclotomic field with restricted ramification. A notable result is Sharifi's theorem, stating that the kernel of the associated Galois representation corresponds to the maximal pro-p extension of the p-th cyclotomic field unramified outside p, at least if $p > 2$ is regular.

In this talk, we present analogous results in the case of a punctured CM elliptic curve over an imaginary quadratic field. Among other things, we give a characterization of the kernel of the associated Galois representation under suitable assumptions. If time permits, we present recent results that further deepen the analogies between genus zero and genus one cases.

[Iha99] Yasutaka IHARA Some arithmetic aspects of Galois actions in the pro-p fundamental group of $\mathbb{P}^1 \setminus \{0, 1, \infty\}$, *Arithmetic fundamental groups and noncommutative algebra (Berkeley, CA, 1999)*, Proc. Sympos. Pure Math., vol. 70, Amer. Math. Soc., Providence, RI, 2002, pp. 247–273.

[Nak95] Hiroaki NAKAMURA, On exterior Galois representations associated with open elliptic curves, J. Math. Sci. Univ. Tokyo 2 (1995), no. 1, 197–231.

[Ish23] Shun ISHII, On the kernels of the pro-p outer Galois representations associated to once-punctured CM elliptic curves (2023). https://arxiv.org/abs/2312.04196

ON A QUILLEN ADJUNCTION BETWEEN THE CATEGORIES OF DIFFERENTIAL GRADED AND SIMPLICIAL COALGEBRAS

Hermann SORÉ (Université Nazi Boni)

The Dold-Kan correspondence is a classic result in homological algebra that establishes an equivalence between the category of graded differential objects and that of simplicial objects in a given abelian category. This naturally raises the question of whether this correspondence is preserved in the non-abelian setting. In this work, we consider the monoidal categories of graded differential vector spaces and simplicial vector spaces and compare their associated categories of comonoids, that is, the category of graded differential coalgebras and the category of simplicial coalgebras. These two categories of comonoids are equipped with model structures established by Getzler and Goerss.

We prove via a counterexample that the Dold-Kan normalisation functor induced at the comonoidal level is not a Quillen equivalence (equivalence of the associated homotopical categories). This deficiency contrasts with the monoidal level, where a Quillen equivalence has been proven.

[Sor19] H. Soré, On a Quillen adjunction between the categories of differential graded and simplicial coalgebras, *J. Homotopy Relat. Struct.* 14, 91–107 (2019). https://doi.org/10.1007/s40062-018-0210-x

Convolution groups of perverse sheaves on abelian varieties

Haohao Liu (IRMA Strasbourg)

Abelian varieties are projective varieties with a group structure. The group law induces a convolution operation for perverse sheaves, which are singular version of local systems. Via the Tannakian formalism, Krämer and Weissauer assign a linear group to each perverse sheaf.

Given a family of perverse sheaves, we show that the convolution group of a very general perverse sheaf in the family is the same as the generic one. We give a geometric application to subvarieties of abelian varieties. This is joint work with Anna Cadoret.

[Liu25] H. Liu, Normality of monodromy group in generic convolution (2025). group https://arxiv.org/abs/2501.14052

[CL25] A. Cadoret, H. Liu. Variation of Tannaka groups of perverse sheaves in family (2025). https://arxiv.org/abs/2505.01716

$M_{0,5}$: Towards the Chabauty-Kim method in higher dimensions

David Jarossay (De Vinci Research Center)

The Chabauty-Kim method provides an approach to the computation of the set of S-integral points of a hyperbolic curve, which is finite by Faltings's theorem: this set is included in sets of common zeroes of certain p-adic Chabauty-Kim functions, defined by using the unipotent fundamental group of X. Works of Dan-Cohen and Wewers and of Dan-Cohen and Corwin develop the Chabauty-Kim method for $\mathcal{M}_{0,4} = \mathbb{P}^1 \setminus \{0, 1, \infty\}$ using mixed Tate motives.

In this talk, we will adapt this framework to the surface $\mathcal{M}_{0,5}$, and we will construct a Chabauty-Kim function on $\mathcal{M}_{0,5}$, thanks to a new method using resultants. This is a joint work with Ishai Dan-Cohen.

[DCJ23] I. Dan-Cohen, D. Jarossay, Towards the Chabauty-Kim method in higher dimensions, Matematika, vol. 69, issue 4, oct. 2023, pp. 1011-1059.

[JLSWZ24] David Jarossay, David T.-B. G. Lilienfeldt, Francesco Maria Saettone, Ariel Weiss and Sa'ar Zehavi. Polylogarithmic motivic Chabauty-Kim for $P^1 \setminus \{0, 1, \infty\}$: the geometric step via resultants (2024: to appear in Algebra & Number Theory). https://arxiv.org/abs/2408.07400

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University, and ENS PSL – see https://ahgt.math.cnrs.fr for workshops, seminars, and list of members.