INSTITUT DE MATHÉMATIQUES DE JUSSIEU - PARIS RIVE GAUCHE, SORBONNE UNIVERSITÉ &
RESEARCH INSTITUTE FOR MATHEMATICAL SCIENCES, KYOTO UNIVERSITY

Paris Arithmetic Homotopic Galois Theory Days 2024

Program and schedule of the workshop

SEPTEMBER 23-24, 2024 - JUSSIEU FRANCE

Org.: B. COLLAS*, RIMS Kyoto University, Japan & E.* LEPAGE*, IMJ-PRG, France*

SCHEDULE

Talks are 75min long. All talks take place at the Campus Jussieu ("Room AA-BB/FRR" is to be read "between tower AA and tower BB, at floor F, room RR"). Some 15 $\sim$ 30 min breaks between talks and extended lunch break allow informal exchanges between participants.

Monday - 23 Sept. ***Room 15-25/502***

9:00 ..... *Welcome*

9:30 ..... B. COLLAS
Oda's problem: arithmetic, braids, and stacks

11:15 ..... Y. TAGUCHI
Mordell-Weil groups over large algebraic extensions of number fields

12:30 ..... LUNCH
Free discussions

14:00 ..... FREE DISCUSSIONS

15:15 ..... E. LEPAGE
Resolution of non-singularities and anabelian applications

16:45 ..... A. MINAMIDE
The outer automorphism groups of the profinite braid groups

Tuesday - Sept. 24 ***Room 15-16/413***

9:00 ..... Y. HOSHI
Anabelian aspects of the Hodge-Tate-ness of Galois representations

10:45 ..... R. WILMS
Valuative compactifications of complex varieties with applications to moduli spaces of curves

12:00 ..... LUNCH
Free discussions.

13:30 ..... F. VERMEULEN
Dimension growth for affine varieties

15:00 ..... A. DUCROS
Skeletons of Berkovich spaces

16:45 ..... K. SAWADA
Families preserving isomorphisms via techniques in anabelian geometry

"Restauration" - a selection

- L'INÉVITABLE, 22 Rue Linné, 75005 Paris - [Google Map]
- BRASSERIE COSMO, 1 Rue des Écoles, 75005 Paris - [Google Map]
- LES BELLES PLANTES, 47 Rue Cuvier, 75005 Paris - [Google Map]
- NARRO, 72 Rue du Cardinal Lemoine, 75005 Paris - [Google Map]
- IPPUDO SAINT-GERMAIN, 714 Rue Grégoire de Tours, 75006 Paris - [Google Map]

A cafeteria at the corner of towers 16-26-25 on the "parvis" provides drinks for the session breaks.

Title & Abstracts

Oda's problem: arithmetic, braids, and stacks

Benjamin Collas (RIMS Kyoto University)

Oda's problem (or prediction), presented at MPI Bonn 1993, deals with the independence of the fixed field of the universal ℓ-monodromy representation of moduli space of curves in terms of the topological data of the curves. This question stands at the intersection of number theory, low dimensional topology (Johnson homomorphism), and arithmetic geometry. It has since acted as a stimulating problem in arithmetic anabelian geometry with multiple approaches – group/Lie theoretic (Matsumoto 96), Galois arithmetic (Ihara-Nakamura 97), and combinatorial anabelian geometric (Hoshi-Mochizuki 11) – to be settled with the joint efforts of Nakamura-Tako-Ueno et al.

After a brief panorama of Oda's problem, this talk will introduce a stack version of Oda's conjecture (and its proof), whose spirit is closer to Oda's original prediction, and which we show, provides another proof of its original schematic version (jt w/ S. Philip JSPS).

[CP24] B. Collas and S. Philip, On Oda's problem and special loci. *RIMS preprint* **1978** (Nov. 2023), pp. 36 http://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1978.pdf

[HM11] Y. Hoshi and S. Mochizuki, On the combinatorial anabelian geometry of nodally non-degenerate outer representations, *Hiroshima Math. J.*, vol. 41, no. 3, pp. 275–342, 2011. DOI:10.32917/hmj/1323700038

[MAT96] M. Matsumoto, Galois representations on profinite braid groups on curves, *J. Reine Angew. Math.*, vol. 474, pp. 160–219, 1996

[TAK14] N. Takao, Some remarks on field towers arising from pro-nilpotent universal monodromy representations, *RIMS Kôkyûroku Bessatsu*, vol. B51, pp. 55–70, 2014 https://repository.kulib.kyoto-u.ac.jp/dspace/bitstream/2433/232891/1/B51-04.pdf

Mordell-Weil groups over large algebraic extensions of number fields

Taguchi Yuichiro (Tokyo Institute of Technology)

Motivated by a conjecture of Frey and Jarden, we present some results on the structure of the Mordell-Weil groups of semiabelian varieties over large algebraic extensions of a number field. We consider two types of algebraic extensions; one is of extensions obtained by adjoining the coordinates of certain points of various semiabelian varieties; the other is of extensions obtained as the fixed subfield in an algebraically closed field by a finite number of automorphisms. Some of such fields turn out to be new examples of Kummer-faithful fields – a notion that originates from anabelian geometry, is a refinement of the notion of sub-p-adic fields, and is suitable for anabelian reconstructions – which are not sub-p-adic.

This talk is based on a joint work with Takuya Asayama [**AT24**].

[AT24] T. Asayama and Y. Taguchi Mordell–Weil groups over large algebraic extensions of fields of characteristic zero, 2024. http://arxiv.org/abs/2408.03495

[FreJ74] G. Frey and M. Jarden, Approximation theory and the rank of abelian varieties over large algebraic fields, *Proc. Lond. Math. Soc.* **28** (1974), 112–128.

[Moc15] S. Mochizuki, Topics in absolute anabelian geometry III: global reconstruction algorithms, *J. Math. Sci. Univ. Tokyo* **22** (2015), 939–1156.

[Oht22] S. Ohtani, Kummer-faithful fields which are not sub-p-adic, *Res. Number Theory* **8** (2022), Paper No. 15, 7 pp.

Resolution of non-singularities and anabelian applications

Emmanuel Lepage (IMJ-PRG)

Given a hyperbolic curve X over an algebraically closed non-archimedean field of mixed characteristics, resolution of non-singularities studies the locus of irreducible components of the special fiber of the stable model of finite étale covers of X. First studied by Tamagawa in 2004, a precise answer to this question was given by Mochizuki and Tsujimura in 2023 for hyperbolic curves defined over a finite extension of $\mathbb{Q}_p$. I will try to explain the main ideas of their proof and give anabelian applications.

[MT23] Shinichi Mochizuki and Shota Tsujimura, Resolution of Nonsingularities, Point-theoreticity, and Metric-admissibility for p-adic Hyperbolic Curves, *RIMS preprint 1974* **1974** (June 2023). https://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1974.pdf

[Lep23] Emmanuel Lepage, Resolution of non-singularities and the absolute anabelian conjecture, *ArXiv preprint* (June 2023). https://arxiv.org/abs/2306.07058

The outer automorphism groups of the profinite braid groups

Minamide Arata (RIMS Kyoto University)

Let $n > 3$ be an integer. Write B_n for the (Artin) braid group on n strings. In 1980's, J. Dyer and E. Grossman proved that any nontrivial outer automorphism of B_n arises from the involution (of order two). In this talk, we will determine the outer automorphism group of the profinite completion $\widehat{B}_n$ of B_n. This is joint work with H. Nakamura.
A key object in this talk is a natural outer action of the Grothendieck-Teichmüller group $\widehat{\mathrm{GT}}$ on $\widehat{B}_n$ (defined by V. Drinfeld and Y. Ihara), where $\widehat{\mathrm{GT}}$ is a profinite group defined as a certain explicit subgroup of the automorphism group of a free profinite group of rank two. One of the most interesting properties of $\widehat{\mathrm{GT}}$ is that there exists a natural injection (which is conjecturally an isomorphism) from the absolute Galois group of the field of rationals into $\widehat{\mathrm{GT}}$.

In our proof, it is important to first focus on a subquotient "$\widehat{\mathcal{P}}_n$" of $\widehat{B}_n$ and apply a computation (which was previously obtained by the speaker in joint work with Y. Hoshi and S. Mochizuki) of the outer automorphism group of $\widehat{\mathcal{P}}_n$. In this talk, we will also explain this computation, in which (combinatorial) anabelian results play important roles.

[HMM22] Y. Hoshi, A. Minamide, and S. Mochizuki, Group-theoreticity of numerical invariants and distinguished subgroups of configuration space groups, *Kodai Math. J.* **45** (2022), pp. 295–348.

[MN22] A. Minamide and H. Nakamura, The automorphism groups of the profinite braid groups, *Amer. J. Math.* **144** (2022), pp. 1159–1176.

Anabelian aspects of the Hodge-Tate-ness of Galois representations

Hoshi Yuichiro (RIMS Kyoto University)

Hodge-Tate-ness is one fundamental notion in the study of p-adic Galois representations. This talk focuses on anabelian aspects of this property.

In particular, I give an explanation of the result that, for an open continuous homomorphism between the absolute Galois groups of p-adic local fields, this homomorphism arises from an embedding of fields if and only if this homomorphism satisfies the condition that the pull-back by this homomorphism of every Hodge-Tate representation is Hodge-Tate. Moreover, I

also give an explanation of the result concerning the existence of an irreducible Hodge-Tate p-adic representation of dimension two of the absolute Galois group of a p-adic local field and a continuous automorphism of the absolute Galois group such that the p-adic Galois representation obtained by pulling back the given p-adic Galois representation by the given continuous automorphism *is not Hodge-Tate*.

[Hsh2013] Yuichiro Hoshi. A note on the geometricity of open homomorphisms between the absolute Galois groups of p-adic local fields, *Kodai Math. J.* **36** (2013), 284-298 DOI: 10.2996/kmj/1372337519

[Hsh2019] Yuichiro Hoshi. Topics in the anabelian geometry of mixed-characteristic local fields, *Hiroshima Math. J.* **49** (2019), 323-398 DOI: 10.32917/hmj/1573787035

[Hsh2024] Yuichiro Hoshi. On intrinsic Hodge-Tate-ness of Galois representations of dimension two, *Kodai Math. J.* **47** (2024), 99-111 doi.org/10.2996/kmj47107

[Mzk1997] Shinichi Mochizuki. A version of the Grothendieck conjecture for p-adic local fields, *Internat. J. Math.* **8** (1997), 499-506 https://doi.org/10.1142/S0129167X97000251

[Mzk2012] Shinichi Mochizuki. Topics in absolute anabelian geometry I: generalities, *(J. Math. Sci. Univ. Tokyo* **19** (2012), 139-242 https://www.ms.u-tokyo.ac.jp/journal/abstract_e/jms190201_e.html

[Ser68] Jean-Pierre Serre. Hodge-Tate decompositions and locally algebraic representations, Appendix in *Abelian l-adic Representations and Elliptic Curves*, W. A. Benjamin, Inc., 1968.

Valuative compactifications of complex varieties with applications to moduli spaces of curves

Robert Wilms* (Université de Caen)

I will talk about valuative compactifications of complex varieties, as recently introduced by Poineau, and their applications to the study of the degeneration of invariants in Arakelov theory. As a motivation, I will first discuss how the study of the degeneration of certain Arakelov invariants applies to arithmetical problems such as the uniform Bogomolov conjecture. Then I will present a new approach to the study of the degeneration of these invariants based on a new canonical compactification of complex varieties constructed in the setting of hybrid Berkovich spaces. After giving the construction of these compactifications, I will apply them to algebraic families of compact Riemann surfaces. I will show that the family of certain Arakelov invariants, such as the canonical Arakelov measure or the Green function, extend to continuous families on the compactifications. This links the degeneration behaviour of these invariants to their non-archimedean counterparts. The degeneration of the canonical Arakelov measure and the Green function has also been computed by de Jong and Amini-Nicolussi using different approaches. Our work provides a different perspective on their results. This is work in progress with Jérôme Poineau.

[Poi24] Jérôme Poineau. Non-archimedean compactifications of complex analytic varieties, 2023. https://poineau.users.lmno.cnrs.fr/Textes/Compactifications.pdf

[AN24] Omid Amini and Noema Nicolussi. Moduli of hybrid curves I: Variations of canonical measures, 2024. https://arxiv.org/abs/2007.07130

[AN22] Omid Amini and Noema Nicolussi. Moduli of hybrid curves II: Tropical and hybrid Laplacians, 2022. https://arxiv.org/abs/2203.12785

*Organizers are grateful to the speaker for his last minute replacement (Formerly Jérôme Poineau (Université de Caen))

Dimension growth for affine varieties

Floris Vermeulen (Lille University & KU Leuven)

Given a projective algebraic variety X over $\mathbb{Q}$, the dimension growth conjecture predicts general upper bounds for the number of points of bounded height on X. It was originally conjectured by Serre, and independently in a uniform way by Heath-Brown. By work of Browning, Heath-Brown and Salberger, uniform dimension growth is now a theorem.

I will give a general overview of dimension growth and explain some ideas of the proof. The main ingredient is the so-called determinant method, which goes back to Bombieri and Pila, and has been succesfully applied to many counting problems. I will then turn to dimension growth for affine varieties, and report on recent work with Raf Cluckers, Pierre Dèbes, Yotam Hendel, and Kien Nguyen.

[CDH+24] Raf Cluckers, Pierre Dèbes, Yotam I. Hendel, Kien Huu Nguyen, Floris Vermeulen. Improvements on dimension growth results and effective Hilbert's irreducibility theorem, *arxiv preprint.* https://arxiv.org/abs/2311.16871

[V24] Floris Vermeulen. Dimension growth for affine varieties, *International Mathematics Research Notices.* **15** (2024), 11464–11483 https://arxiv.org/abs/2311.05433

Skeletons of Berkovich spaces

Antoine Ducros (IMJ-PRG)

This will be a survey talk around the notion of a skeleton of a Berkovich analytic space. Skeletons are subsets of analytic spaces that inherit a natural piecewise linear structure. They play a key role for describing the homotopy type of analytic spaces and are often related to deep arithmetic properties of algebraic varieties (semi-stable reductions, Neron models...). I plan to give a lot of examples and to present the main results of the theory, including some very recent ones, and explain which tools are used in the proofs without entering into technical details.

[Duc2016] Antoine Ducros About Hrushovski and Loeser's work on the homotopy type of Berkovich spaces, in *Nonarchimedean and tropical geometry. Based on two Simons symposia, Island of St. John, March 31 – April 6, 2013 and Puerto Rico, February 1–7, 2015*, 99-131 https://webusers.imj-prg.fr/~antoine.ducros/Simons-Lectures-v3.pdf

[Duc2012] Antoine Ducros Espaces de Berkovich, polytopes, squelettes et théorie des modèles, *Confluentes Math.* **4** (2012), 1250007, 57 pages. Erratum in *Confluentes Math.* **5** (2013), 43-44. https://webusers.imj-prg.fr/~antoine.ducros/polytopes_et_squelettes.pdf et https://webusers.imj-prg.fr/~antoine.ducros/erratum-squelettes.pdf

[DHLY2024] Antoine Ducros, Ehud Hrushovski, François Loeser and Jinhe Ye, Tropical functions on a skeleton, *Journal de l'École polytechnique* **11** (2024), 613-654 https://jep.centre-mersenne.org/item/10.5802/jep.261.pdf

Families preserving isomorphisms via techniques in anabelian geometry

Sawada Koichiro (RIMS Kyoto University)

For an isomorphism between closed subgroups of a profinite group G, if the image of any pro-cyclic subgroup I via this isomorphism is conjugate in G to I, then we shall say that this isomorphism is *"families preserving"*.

Jarden and Ritter showed that, for a certain type of profinite group G – that includes the absolute Galois group of a p-adic local field and the étale fundamental group of hyperbolic curves–, every normal automorphism of G is inner. Their proof was divided into two steps:

first showing that every normal automorphism is "families preserving" (in G), and then showing that every "families preserving" automorphism is inner.

n this talk, we discuss, from an anabelian geometrical point of view, whether a "families preserving" isomorphism between closed subgroups of a profinite group, such as the absolute Galois group of a certain field (Hilbertian field, Henselian discrete valuation field) or its quotient, is induced from an inner automorphism. This is a joint work with Arata Minamide and Shota Tsujimura.

[JR80] M. Jarden and J. Ritter, Normal automorphisms of absolute Galois groups of $\mathfrak{p}$-adic fields,*Duke Math. J.* 47 (1980), no. 1, 47–56. https://doi.org/10.1215/S0012-7094-80-04705-5

[MST24] A. Minamide, K. Sawada, S. Tsujimura. Families preserving isomorphisms via techniques in anabelian geometry (2024: in preparation, title to be confirmed).

[MST22] A. Minamide, K. Sawada, S. Tsujimura. On Generalizations of Anabelian Group-theoretic Properties, *RIMS preprint* **1965** (Aug. 2022), 25 pp. https://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1965.pdf

This workshop is part of the "Arithmetic & Homotopic Galois Theory" project, supported by the CNRS-RIMS AHGT International Research Network between Lille University, RIMS Kyoto University, and ENS PSL – see https://ahgt.math.cnrs.fr for workshops, seminars, and list of members.