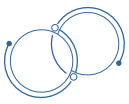
ENS-RIMS - 20-24 March 2023

Arithmetic & Homotopic Galois Theory 2023

Programme & Schedule of the Workshop



Arithmetic & Homotopic Galois Theory

A France-Japan CNRS International Network

Organizing Committee
B. Collas, RIMS Kyoto, Japan
P. Dèbes, Université de Lille, France
Y. Hoshi, RIMS Kyoto, Japan
A. Mézard, ENS-PSL, France

Title and Abstracts

One or two remarks on the Tate conjecture

Yves André (Sorbonne Université)

The Tate conjecture has two parts: i) Tate classes are linear combination of algebraic classes, ii) Galois representations attached to smooth projective varieties are semisimple. B. Moonen proved that i) implies ii) in characteristic 0, using p-adic Hodge theory.

We'll show that an unconditional result lies behind this implication: the "observability" of arithmetic monodromy groups of geometric origin (in any characteristic) - which leads to a sharpening of Moonen's result.

Time permitting, we'll also discuss another aspect of the Tate conjecture related to the transcendence of p-adic periods.

Bounds for rational points on algebraic curves and dimension growth

Raf Cluckers (Lille University & Leuven University)

I will present new work with Binyamini and Novikov on a question by Salberger, about (optimal) bounds for the number of rational points of bounded height on algebraic curves, namely, for height up to B on an integral curve of degree d, the upper bound is $d^2B^{2/d}$ times a poly log B factor. In this bound, the d^2 factor is new and the bound is optimal (apart from the poly log B factor). This leads to corresponding improvements and simplifications to so-called dimension growth results. The dimension growth conjecture was coined by Browning but initially raized as a question by Heath-Brown and Serre, and almost all degrees are (optimally) known by now, except most importantly the uniformity in degree 3. In work in progress with Dèbes, Hendel, Nguyen and Vermeulen we present a question on curves that would lead to further simplifications, strengthenings, and generalizations of the dimension growth results, in particular in degree 3. In this approach we generalize the affine situation (unconditionally), using a new effective, higher dimensional variant of Hilbert's irreducibility theorem.

Prospects in Arithmetic and Homotopy Galois Theory

Benjamin Collas (RIMS, Kyoto University)

The development of Arithmetic and Homotopy Galois Theory (AHGT), in all its variants (such as the inverse Galois problem, Noether program, Ihara program, anabelian geometry, and motivic theory), has relied on Grothendieck's original insight: the reconciliation of number theory and geometry, and the encounter of the categorical formalism with some seeding theories, with, as a result, the identification of new seminal research frontiers.

These principles, with the implicit inclusion of under-exploited techniques from Grothendieck's school – such as Artin étale simplicial type, Quillen model category, Verdier-Illusie derived formalism, or Deligne unifying approach – and the growing development of the Japanese anabelian arithmetic schools, have recently been reignited. With new encounters and new shifts of perspective it results in the premise of new geometries, which in particular clarify and soften the boundary between various mathematical realms (homological vs homotopical,...)

The goal of this talk is to follow these lines of approach to sketch a structured panorama of AHGT. A skeleton of central themes should appear that will motivate the future construction of a coherent network of Conjectures in Arithmetic Geometry (CAGE). As a support, concrete IRN activities will be presented.

> We neither can nor want to lay out a route for the future development of our science; this would be a futile task, indeed it would be a ridiculous enterprise, for the great mathematicians of the future, like those of the past, will flee from the beaten track. They will solve the great problems which we shall bequeath to them, through unexpected connections, which our imagination will not have succeeded in discovering, and by looking at them in a new light.
>
> A. Weil, The Future of Mathematics, 1950.

Hurwitz Spaces

Pierre Dèbes (Lille University)

Hurwitz spaces are moduli spaces of covers of the projective line. Their topological origin made them very useful to prove the irreducibility of the moduli space of curves of given genus. I will focus on the arithmetic part of their history, which started in the seventies. After reviewing the earlier achievements, notably towards the Inverse Galois Problem, I will emphasize a series of results from the last ten years, some very recent and all based on the notion of product of components of Hurwitz spaces.

Noether's and Grunwald's problems: rationality, approximation and obstructions

Cyril Demarche (Sorbonne University)

The Noether problem asks whether the subfield of invariant functions in the function field of a faithful representation of a finite group is a rational extension of the base field. A positive answer to this question leads to a positive answer to the inverse Galois problem over global fields, in a strong form where one can prescribe the local Galois group at a finite number of places (the so-called Grunwald's problem). We will survey variants of the Noether rationality problem and of the Grunwald weak approximation question, explaining certain obstructions to rationality (e.g. unramified cohomology) or to weak approximation (e.g. Brauer-Manin obstruction). We will mention recent progresses on these questions, including for instance the solution to the Grunwald problem for supersolvable groups or counting results on points of bounded height on varieties associated to the inverse Galois problem.

A geometric interpretation of double shuffle relations between multiple zeta values

Benjamin Enriquez (Strasbourg University)

The "associator" and "double shuffle" relations between multiple zeta values give rise to two torsors (due respectively to Drinfeld and Racinet), which both contain the torsor of mixed Tate motives over $\mathbb{Z}$. Following ideas of Deligne and Terasoma, we propose a geometric interpretation of the "double shuffle" torsor. This is applied to the solution of two problems: (a) making explicit the bitorsor structure underlying the double shuffle torsor; (b) proving (independently of Furusho's 2011 proof) the inclusion of the associator torsor in the double shuffle one.

This is joint work with H. Furusho.

Combinatorial Arithmetic Geometry

Geoffroy Horel (Sorbonne Paris Nord University)

I will explain the Galois action on the moduli spaces of curves and variants thereof from a combinatorial and homotopical perspective. This circle of ideas originates in the work of Drinfel'd and comes from seeing these moduli spaces as capturing the universal operations on braided monoidal categories. I will mention some consequences in deformation theory and low dimensional topology.

Tempered fundamental group

Emmanuel Lepage (Sorbonne University)

The goal of this talk is to review results and applications of the tempered fundamental group in anabelian geometry. The tempered fundamental group of a variety over a non-archimedean field, defined by Y. André, classifies a category of covers which contain finite covers, but also some infinite covers such as the uniformization of Tate curves. Whereas its profinite completion coincides with the profinite étale fundamental group, the tempered fundamental group has interesting anabelian features even over algebraically closed fields of mixed characteristics.

Inter-universal Teichmüller Theory as an Anabelian Gateway to Diophantine Geometry and Analytic Number Theory

Shinichi Mochizuki (RIMS, Kyoto University)

One question that is frequently asked concerning *inter-universal Teichmüller theory* (IUT) is the following:

Why/how does IUT allow one to apply *anabelian geometry* to prove *diophantine* results?

In this talk, we address this question from various points of view. First, we discuss the fundamental framework underlying the relationship established by IUT between anabelian geometry, on the one hand, and diophantine geometry/analytic number theory, on the other. This discussion centers around the N-th power map on a subring of a field and the difference between regarding a group as a *Galois group* and as an *abstract group* that is not equipped with an embedding into the automorphism group of a field. Here, we emphasize that this discussion is *entirely elementary* and only assumes a knowledge of *groups/monoids, rings, fields*, and the elementary geometry surrounding the *projective line/Riemann sphere*. We then proceed to survey recent developments (work in progress) in IUT, namely, the *Galois-orbit version* of IUT, which has new applications to the *Section Conjecture* (in anabelian geometry) and the *nonexistence of Siegel zeroes of certain Dirichlet L-functions*. The application to the Section Conjecture is interesting in that it exhibits and reconfirms the *essentially anabelian content of IUT*, i.e., as a *theory based on anabelian geometry that is applied to prove new results in anabelian geometry*. On the other hand, these recent applications, taken together with the original application of IUT to the ABC/Szpiro/Vojta Conjectures, are also noteworthy in that they may be regarded as a striking example of Poincaré's famous quote to the effect that

"mathematics is the art of giving the same name to different things".

That is to say, the *common name "IUT"* that may be regarded as describing, in essence, a *single mathematical phenomenon* that manifests itself, depending on relatively inessential (!) differences of context, as various (at first glance, unrelated!) *diverse phenomena in anabelian geometry, diophantine geometry*, and *analytic number theory*. The relationship with Poincaré's famous quote is also fascinating in that it was apparently motivated by various mathematical observations on the part of Poincaré concerning the similarities between *transformation group symmetries of modular functions such as theta functions* and *symmetry groups of the hyperbolic geometry of the upper half-plane* — all of which are topics (cf. the discussion above of Galois groups versus abstract groups!) that bear a profound relationship to IUT.

A two-parameter family of tropical ultra-discretizations of an elliptic curve

Hiroaki Nakamura (Osaka University)

In this talk, I will discuss a certain two-parameter family of ellitpic curves on the plane to provide explicitly computed tropical curves corresponding to their degeneration. Applying the theta uniformization with the method of ultra-discretization by Kajiwara-Kaneko-Nobe-Tsuda, we give a formula for the coordinate functions that traces the cycle part of the tropical elliptic curve. We also illustrate how to recover the whole part of the tropical curve as a quotient of the Bruhat-Tits tree after Speyer's algebraic approach.

This is a joint work with Rani Sasmita Tarmidi.

Reconstruction of the residue field of a local field from its Galois group

Yuichiro Taguchi (Tokyo Institute of Technology)

I will explain my attempt to reconstruct the (additive structure of the) residue field of a non-Archimedean local field K from a topological group isomorphic to the absolute Galois group of K.

Recent development in anabelian geometry of global fields

Akio Tamagawa (RIMS, Kyoto University)

The anabelian geometry of global fields – number fields and global function fields (i.e. function fields of one variable over finite fields) – was established very nicely in the 1970s, long before Grothendieck proposed the philosophy of anabelian geometry, and is now well known in the names of the Neukirch-Uchida theorem and Uchida's theorem. In this talk, we will survey recent development in anabelian geometry of global fields, where the absolute Galois groups are replaced by various quotients.

p-adic Teichmüller theory and the counting problem of dormant opers

Yasuhiro Wakabayashi (Osaka University)

p-adic Teichmüller theory is a branch of arithmetic geometry that studies the uniformization of hyperbolic curves in positive and mixed characteristic. The key idea of this theory is to endow hyperbolic curves with certain flat bundles, called indigenous bundles (or more generally, opers), and to investigate the resulting moduli space using tools from p-adic and logarithmic geometry. The study of the moduli space of such bundles has many connections in mathematics, including the combinatorics of convex polytopes, the representation theory of affine Lie algebras, and the Gromov-Witten theory of relative Grassmann varieties. In this talk, I would like to give a brief introduction to p-adic Teichmüller theory and explain the related counting problem of indigenous bundles based on the connections listed above.

Geometry related with the absolute Galois group of $\mathbb{Q}_p$

Seidai Yasuda (Osaka University)

The main ingredient of this talk is an exposition of my joint work with Go Yamashita on the construction of integral lattices of some 2-dimensional crystalline representations of the absolute Galois group of the field $\mathbb{Q}_p$ of p-adic numbers. Besides, I would also like to talk about some related or non-related results and problems on the geometry related with the absolute Galois group of $\mathbb{Q}_p$.

Schedule of the week

Format of the talks is 45 minutes of *presentation* followed by 15 minutes of *discussions* (then 15 minutes break). `Zoom` attendance follows the usual best practice: **video ON and microphone OFF**. Exchanges during the talk will be led within the `corresponding Discord channel`.

Monday — March, the 20th

8:00 (FR) ..... ***Welcoming*** ..... **(JP) 16:00**
Welcoming participants at ENS and RIMS.

8:30 ..... ***Opening words*** **16:30**
Professor K. Ono (RIMS Director) & Professor J.S. Dhersin (CNRS INSMI Scientific dep. Director).

8:45 ..... Collas **16:45**
Prospects in Arithmetic and Homotopy Galois Theory.

9:45 ..... André **17:45**
One or two remarks on the Tate conjecture.

Break 15min

10:45 ..... Tamagawa **18:45**
Recent development in anabelian geometry of global fields.

11:45 ..... Taguchi **19:45**
Reconstruction of the residue field of a local field from its Galois group.

12:45 ..... ***Free Discussion*** **20:45**

Tuesday — March, the 21th

8:30 ..... Dèbes **16:30**
Hurwitz Spaces.

9:45 ..... Lepage **17:45**
Tempered fundamental group.

Break 15min

11:15 ..... Horel **19:15**
Combinatorial Arithmetic Geometry.

12:15 ..... ***Free Discussion*** **20:15**

Wednesday — March, the 22nd

9:00 ..... Cluckers **17:00**
Bounds for rational points on algebraic curves and dimension growth.

10:15 ..... Mochizuki **18:15**
Inter-universal Teichmüller Theory as an Anabelian Gateway to Diophantine Geometry and Analytic Number Theory.

11:15 ..... ***Free Discussion*** **19:15**

Social Dinner (JP) **20:30**

Thursday — March, the 23th

8:30 ..... Yasuda **16:30**
Geometry related with the absolute Galois group of $\mathbb{Q}_p$.

9:45 ..... Wakabayashi **17:45**
p-adic Teichmüller theory and the counting problem of dormant opers.

Break 15min

11:15 ..... Enriquez **19:15**
A geometric interpretation of double shuffle relations between multiple zeta values.

12:15 ..... ***Free Discussion*** **20:15**

Friday — March, the 24th

9:00 ..... Demarche **17:00**
Noether's and Grunwald's problems: rationality, approximation and obstructions.

10:15 ..... Nakamura **18:15**
A two-parameter family of tropical ultradiscretizations of an elliptic curve.

11:15 ..... ***Closing words*** **19:15**

Rooms. At RIMS, main building, seminar room 110; At ENS, room W (Wed. room R3).

Free Discussion. At RIMS and ENS, a common room will be available every day starting 15:00 JP, 8:30 FR for one-to-one exchanges, impromptus complementary talks, and personal work (with Temiyage, blackboard, and Eduroam Wi-Fi access).

Vernal Equinox Day: March 21st is bank holiday in Japan – 春分の日 (shunbun no hi) – access to the RIMS building will be provided by one of the organizers.

The AHGT23 Discord channels: Entry point to the workshop is given by a private AHGT23 Discord workspace organized in `#channels` that **(1)** provides the latest schedule and the links to each zoom talk (and their recordings for US participants), **(2)** gathers some additional material, and **(3)** offers a place for email-chat like live and asynchronous discussions[1].

[1] Access is given to AHGT23 participants only; online content will be deleted after one month.

List of Participants

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Online only [43 participants]. For (an updated) complete list of on-line participants including their professional homepage (and thus a way of contact), see the webpage of the AHGT23 workshop.